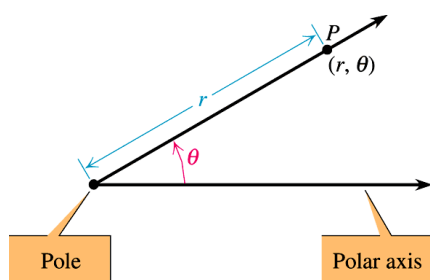


## SM3H 9.6 Polar Equations Notes



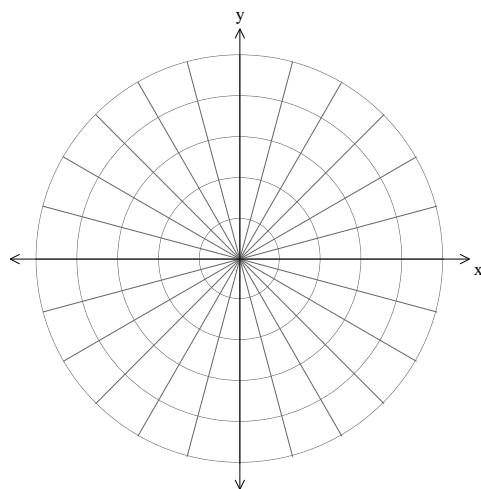
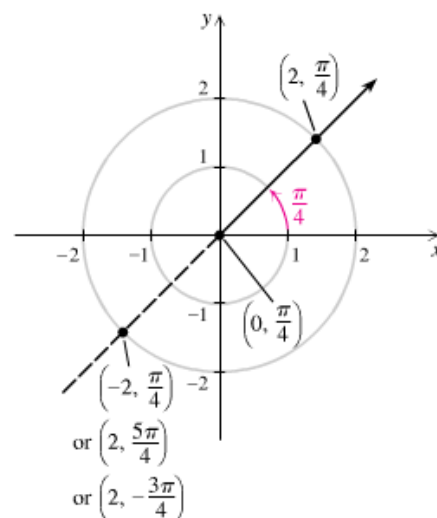
The polar coordinate system is based on a fixed point called the **pole** and a fixed axis called the **polar axis**. Points are represented by ordered pairs in the form  $(r, \theta)$ , where  $r$  is the **directed distance** from the pole and  $\theta$  is an angle whose initial side is the polar axis and whose terminal side contains the point. Typically, we choose the origin as the pole and the positive  $x$ -axis as the polar axis.

\*To graph  $(-r, \theta)$ , you move in the opposite direction you would move to graph  $(r, \theta)$ .

Polar coordinates are not unique. The points  $(-2, \frac{\pi}{4})$ ,  $(2, \frac{5\pi}{4})$ , and  $(2, -\frac{3\pi}{4})$  all name the same point.

**Examples:** Plot the points whose polar coordinates are given.

$A(3, \frac{\pi}{3})$ ,  $B(-1, \frac{\pi}{6})$ ,  $C(2, -\frac{7\pi}{4})$ ,  $D(-5, -\frac{3\pi}{4})$ ,  $E(4, \frac{\pi}{2})$ ,  $F(-3, \frac{2\pi}{3})$



### Polar-Rectangular Conversion Rules

- To convert  $(r, \theta)$  to rectangular coordinates  $(x, y)$ , use  $x = r \cos \theta$  and  $y = r \sin \theta$ .
- To convert  $(x, y)$  to polar coordinates  $(r, \theta)$ , use  $r = \sqrt{x^2 + y^2}$  and any angle  $\theta$  in standard position whose terminal side contains  $(x, y)$ .

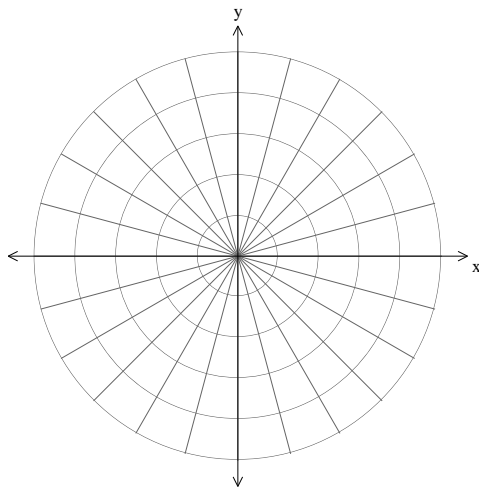
**Examples:**

- a) Convert  $(3, 45^\circ)$  to rectangular coordinates.      b) Convert  $(-2, 2\sqrt{3})$  to polar coordinates.

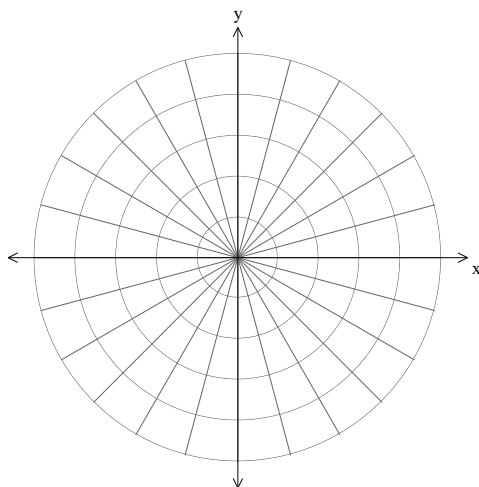
### Graphing Polar Equations

**Examples:** Sketch the graphs of the following:

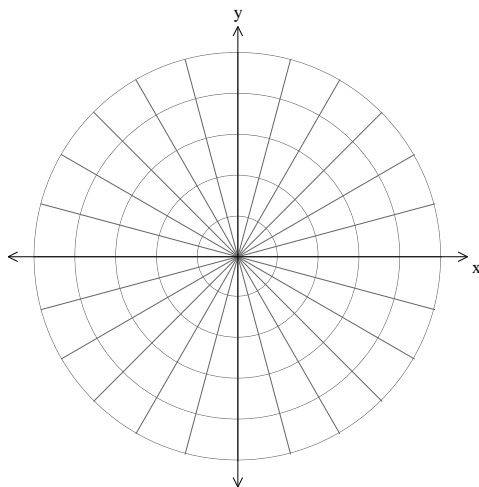
a)  $r = 4 \sin \theta$



b)  $r = \cos(2\theta)$



c)  $r = 3 + 4 \sin \theta$



**Examples:** Convert equations from polar to rectangular form.

a) Convert  $r = 3\sin \theta$  to a rectangular equation.

b) Convert  $r = \frac{4}{1 + \sin \theta}$  to a rectangular equation.

**Examples:** Convert equations from rectangular to polar form.

c) Convert  $y = -2x + 5$  to a polar equation.

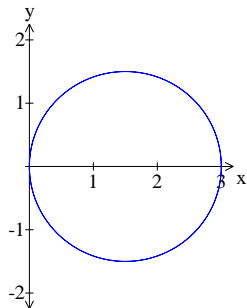
d) Convert  $x^2 + (y - 1)^2 = 1$  to a polar equation.

1) Lines through the origin are of the form  $\theta = \alpha$ .

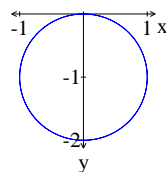
Vertical lines are of the form  $r \cos \theta = a$ .

Horizontal lines are of the form  $r \sin \theta = a$ .

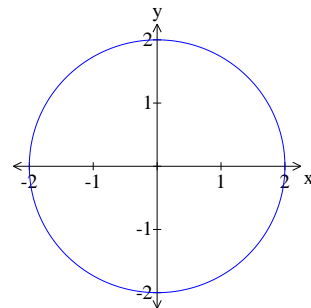
2) Circles come in three forms:  $r = a \cos \theta$ ,  $r = a \sin \theta$ , and  $r = a$ .



$$r = 3 \cos \theta$$



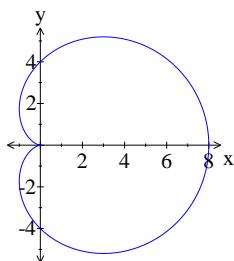
$$r = -2 \sin \theta$$



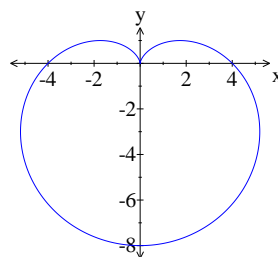
$$r = 2$$

3) Cardioids have the form  $r = a \pm a \cos \theta$  or  $r = a \pm a \sin \theta$ .

Cardioids pass through the pole.



$$r = 4 + 4 \cos \theta$$



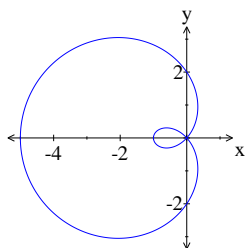
$$r = 4 - 4 \sin \theta$$

4) Limaçons have the form  $r = a \pm b \cos \theta$  or  $r = a \pm b \sin \theta$ .

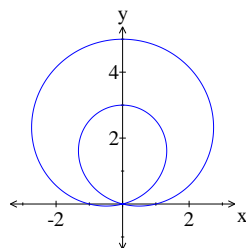
Limaçons have an inner loop if  $0 < a < b$  and have no inner loop if  $0 < b < a$ .

The graph of a limaçon with an inner loop passes through the pole twice.

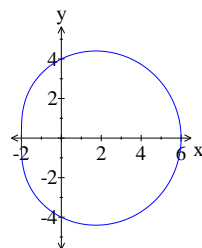
The graph of a limaçon with no inner loop does not pass through the pole.



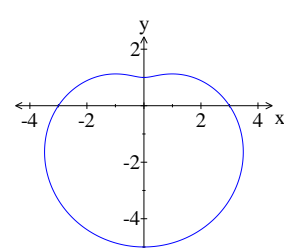
$$r = 2 - 3 \cos \theta$$



$$r = 1 + 4 \sin \theta$$

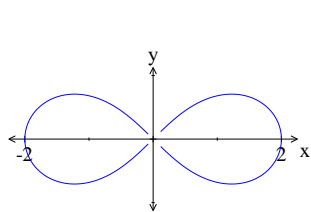


$$r = 4 + 2 \cos \theta$$

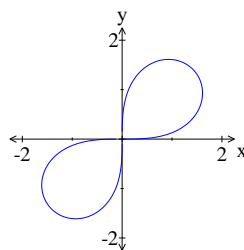


$$r = 3 - 2 \sin \theta$$

5) Lemniscates have the form  $r^2 = a^2 \cos(2\theta)$  or  $r^2 = a^2 \sin(2\theta)$ .



$$r^2 = 4 \cos(2\theta)$$

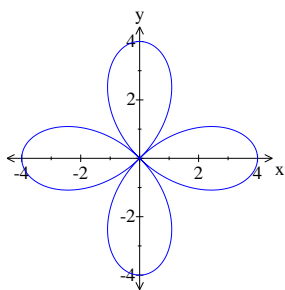


$$r^2 = 4 \sin(2\theta)$$

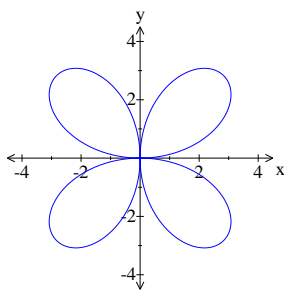
6) Roses have the form  $r = a \cos(n\theta) + b$  and  $r = a \sin(n\theta) + b$ .

If  $n$  is even, there are  $2n$  loops in the rose.

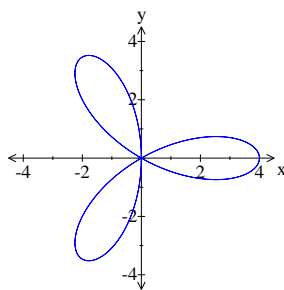
If  $n$  is odd, there are  $n$  loops in the rose.



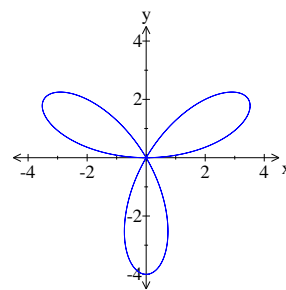
$$r = 4 \cos(2\theta)$$



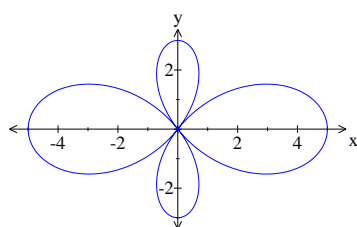
$$r = 4 \sin(2\theta)$$



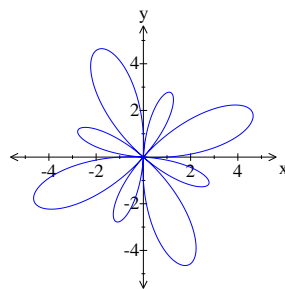
$$r = 4 \cos(3\theta)$$



$$r = 4 \sin(3\theta)$$



$$r = 4 \cos(2\theta) + 1$$



$$r = 4 \sin(4\theta) + 1$$